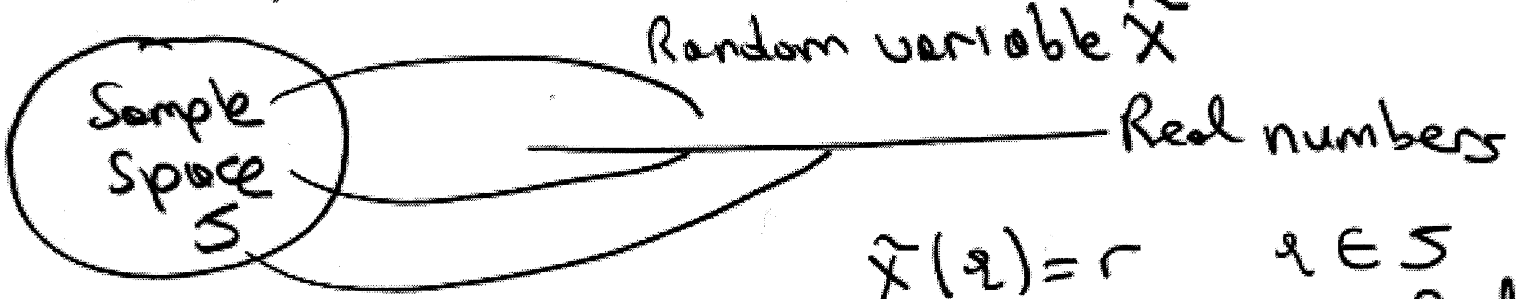
Random Variable:

Random variable is a real valued function of the experimental outcomes.



$\tilde{X}(\xi) = r$ $\quad \xi \in S$

$r \rightarrow$ Real Number

Ex:

$S = \{h, t\} \rightarrow$ coin toss

$\tilde{X}(h) = 3.14$

$\tilde{X}(t) = 6.1$

$\tilde{X} \rightarrow$ A random variable defined for coin toss experiment

Ex:

Experiment: 5 tosses of a coint.

Sample space: $S = \{hhhhh, hhhht, \ldots\}$

$N(S) = 32$ points in S, $\eta \in S$, $\eta \rightarrow$ simple event.

$X \rightarrow$ Random variable.

$X(\eta)$ = Number of heads in the sequence η

$\eta \in S$

Ex:

Two rolls of a die. $S = \{11, \underbrace{12}_{\eta \in S}, 13, 14 \ldots\} \rightarrow 36$ points

Random variables

a) The sum of the two rolls

b) The number of sixes in the two rolls

Notation:

$\{\zeta \mid \tilde{x}(\zeta) = x\}$ → The set of simple events ζ satisfying $\tilde{x}(\zeta) = x$

Ex: Toss of two coin.

$S = \{hh, ht, th, tt\}$

Random variable $\tilde{x}$ is defined on $\zeta \in S$ as

$\tilde{x}(hh) = 1 \quad \tilde{x}(tt) = 1 \quad \tilde{x}(ht) = 2 \quad \tilde{x}(th) = 4$

Then $\{\zeta \mid \tilde{x}(\zeta) = 1\} = \{hh, tt\}$

$\{\zeta \mid \tilde{x}(\zeta) = 2\} = \{ht\}$

Short notation for $\{q \mid \hat{x}(q) = x\}$

$$\{q \mid \hat{x}(q) = x\} = \{\hat{x} = x\}$$

$\{\hat{x} = x\} \rightarrow$ will be used for $\{q \mid \hat{x}(q) = x\}$ for simplicity

Probability Mass Function:

$p(x) = p\{\hat{x} = x\}$

Ex: Toss of two coin.

$S = \{hh, ht, th, tt\}$

$\hat{x} \to$ Random variable.

$\hat{x}(hh) = 1$ $\quad \hat{x}(ht) = 2$

$\hat{x}(tt) = 1$ $\quad \hat{x}(th) = 4$

$p(x) = p(\hat{x} = x) \to p(1) = p(\hat{x} = 1)$

$= p(\{hh\}, \{tt\}\}$

$= p(hh) + p(tt)\}$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$p(2) = p(\hat{x} = 2)$

$= p(ht)$

$= 1/4$

$p(3) = p(\hat{x} = 3)$

$= p(\phi)$

$= 0$

$p(4) = p(\hat{x} = 4)$

$= p(th)$

$= \frac{1}{4}$

Cumulative Distribution Function of Random Variable $\tilde{x}$

$\tilde{x} \rightarrow$ Random Variable (R.V.)

$\{z \mid \tilde{x}(z) \leq x\} \rightarrow$ set of simple events z such that they satisfy $\tilde{x}(z) \leq x$ $\quad x \in$ Real Numbers

Short Notation for $\{z \mid \tilde{x}(z) \leq x\} \Rightarrow \{\tilde{x} \leq x\}$.

Ex: $S=\{hh, ht, th, tt\}$

$\hat{x}(hh)=1$
$\hat{x}(ht)=2$
$\hat{x}(th)=3$
$\hat{x}(tt)=5$

then $\{q \mid \hat{x}(q) \leq 1\} = \{hh\}$

$\{q \mid \hat{x}(q) \leq 3\} = \{hh, ht, th\}$

OR

$\{\hat{x} \leq 3\} = \{hh, ht, th\}$

Distribution function of R.V $\hat{x}$ is defined

as $F(x) = P(\hat{x} \leq x)$

Ex: $S=\{hh, ht, th, tt\}$ $\hat{x}(hh)=2$ $\hat{x}(ht)=3$ $\hat{x}(th)=5$
$\hat{x}(tt)=1$

Compute $F(1)$, $F(2)$, $F(3)$ for $\hat{x}$ $\qquad$ $F(x) \to$ Distribution function of $\hat{x}$.

Sln: $F(x)=P(\hat{x} \leq x) \to F(1)=P(\hat{x} \leq 1)=P(tt)=\frac{1}{4}$

$F(2)=P(\hat{x} \leq 2)=P(\{tt\},\{hh\})=P(tt)+P(hh)$

$=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

$$F(3) = P(\tilde{X} \le 3) = P(\{hh\}, \{ht\}, \{tt\})$$
$$= p(hh) + p(ht) + p(tt)$$
$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}.$$

Example: $S = \{hh, ht, th, tt\}$ $\tilde{X} \to$ R.V. on S

Draw $F(x)$ of $\tilde{X}$. (i.e., draw distribution function of R.V. $\tilde{X}$)

$\tilde{X}(hh) = 2$

$\tilde{X}(ht) = 4$

$\tilde{X}(th) = 5$

$\tilde{X}(tt) = 11.$

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S/n: $F(x) = P(\tilde{x} \le x)$

$P(\tilde{x} \le x) = P(\phi) = 0 \qquad x < 2$

$P(\tilde{x} \le x) = P(hh) = \frac{1}{4} \qquad 2 \le x < 4$

$P(\tilde{x} \le x) = P(\{hh\}, \{ht\}) \qquad 4 \le x < 5$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(\tilde{x} \le x) = P(hh, ht, th) \qquad 5 \le x < 11$

$= \frac{3}{4}$

$P(\tilde{x} \le x) = P(hh, ht, th, tt) \qquad 11 \le x$

$= P(S)$

$= 1$