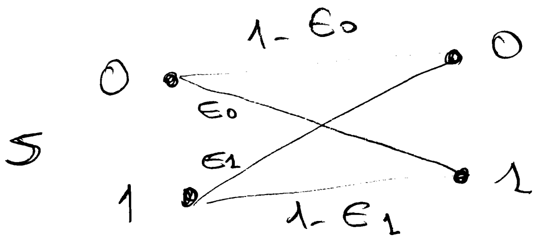
ECE 307 HW#2 Answers

(1)



s → transmitted symbol

r → received symbol

$\text{Prob}(s=0) = p \qquad \text{Prob}(s=1) = 1-p$

$\text{Prob}(r=0 \mid s=0) = 1-\epsilon_0 \qquad \text{Prob}(r=1 \mid s=1) = 1-\epsilon_1$

The prob. of receiving 0 is received given that 0 is transmitted

$\text{Prob}(r=0 \mid s=1) = \epsilon_1$

$\text{Prob}(r=1 \mid s=0) = \epsilon_0$

a) $A = \{$ kth symbol is received correctly $\}$

$S_0 = \{$ transmit bit-0 $\}$

$S_1 = \{$ transmit bit-1 $\}$

$R_0 = \{$ receive bit-0 $\}$

$R_1 = \{$ receive bit-1 $\}$

$$P(A) = P(S_0 \cap R_0) + P(S_1 \cap R_1)$$
$$= P(R_0 \mid S_0) \cdot P(S_0) + P(R_1 \mid S_1) P(S_1)$$
$$= (1-\epsilon_0)\, p + (1-\epsilon_1) \cdot (1-p)$$

b) 1011 → transmitted sequence

1011 → correctly received sequence

$$P(r=1 \mid s=1)\, P(r=0 \mid s=0)\, P(r=1 \mid s=1)\, P(r=1 \mid s=1)$$
$$(1-\epsilon_1)(1-\epsilon_0)(1-\epsilon_1)(1-\epsilon_1)$$
$$= (1-\epsilon_0)(1-\epsilon_1)^3$$

(2) A & B are independent events

i.e, $P(A \cap B) = P(A) \cdot P(B)$

a) The events A & B^c are independent

$A = A \cap (B \cup B^c) \rightarrow A = (A \cap B) \cup (A \cap B^c)$ disjoint events

$P(A) = P(A \cap B) + P(A \cap B^c)$

$\rightarrow P(A \cap B^c) = P(A) - \underbrace{P(A \cap B)}_{= P(A) \cdot P(B)}$

$P(A \cap B^c) = P(A)\underbrace{(1 - P(B))}_{P(B^c)}$

$P(A \cap B^c) = P(A) P(B^c)$

b) The events A^c & B^c are independent

It can be shown as in step a) $P(A^c \cap B) = P(A^c) P(B)$

$B = B \cap (A \cup A^c) \longrightarrow P(A^c \cap B) = P(A^c) P(B)$

Then using $A^c = A^c \cap (B \cup B^c) \longrightarrow P(A^c \cap B^c) = P(A^c) P(B^c)$

(3) A, B, C independent events, prove that A & B are conditionally independent given C

$P(A \cap B | C) = \dfrac{P(A \cap B \cap C)}{P(C)} = \dfrac{P(A) P(B) P(C)}{P(C)}$

$= P(A) P(B) = P(A|C) P(B|C)$

so A & B are conditionally independent given C

4

52 cards in a deck

13 of them are RED 13 of them are BLACK

13 of them are BLUE

13 of them are YELLOW

Three cards are drawn without replacement

What is the prob that none of the cards is RED?

$\binom{52}{3}$ → Sample Space

13 BLUE + 13 YELLOW + 13 BLACK → 39 CARDS

$$\frac{\binom{39}{3}}{\binom{52}{3}}$$ → the prob. that none of the cards is RED