Examples:

Ex: Write $A \cup B$ as unions of two mutually exclusive events

Sln: For any event A we know that

$S \cap A = A$ $\quad S \to$ Sample space

and $A \cup A^c = S$ $\quad A^c \to$ complementary event of A

then
$$A \cup B = S \cap (A \cup B)$$
$$= (A \cup A^c) \cap (A \cup B)$$
$$= \underbrace{A}_{X} \cup \underbrace{(A^c \cap B)}_{Y}$$

It is clear that $X \cap Y = \phi$

Since $\underbrace{A \cap A^c}_{\phi} \cap B = \phi$

Then we can say that A and $A^c \cap B$ are mutually exclusive events

(2) Ex:

Show that $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

Sln:

$A \cup B = \underbrace{A \cup A^c B}$

mutually exclusive events

$B = \underbrace{AB \cup A^c B}$

mutually exclusive events

$p(A \cup B) = p(A) + p(A^c B)$ ← put into

$p(B) = p(AB) + p(A^c B) \rightarrow p(A^c B) = p(B) - p(AB)$

$\rightarrow p(A \cup B) = p(A) + p(B) - p(A \cap B)$

Note: $S = \{e_1, e_2, e_3 \dots e_N\}$ → sample space

e_i → simple outcome

If the experiment is a fair one

and $p(e_i) = 1/N$ → prob. of simple outcomes are all equal to $1/N$

then $A = \{a_1, \dots a_K\}$ → an event

i.e; subset of S

then $p(A) = \dfrac{N(A)}{N(S)}$ → prob. of event A

$= \dfrac{K}{N}$

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If $p(q_i) \neq \frac{1}{N}$ for $\forall i$

then
$$p(A) = p(u_1) + p(u_2) + \dots p(u_K)$$

Ex:

$S = \{1,2,3,4,5\}$

$p(1) = a \quad p(2) = 2a \quad p(3) = a \quad p(4) = 2a \quad p(5) = 4a$

$A = \{1,3,4\} \quad B = \{2,4,5\} \quad C = \{1,4\}$

$p(A) = ? \quad p(B) = ? \quad p(C) = ?$

Sln:

$p(S) = 1 \quad p(S) = p(1) + p(2) + p(3) + p(4) + p(5)$

$1 = a + 2a + a + 2a + 4a$

$1 = 10a \longrightarrow a = 1/10$

Then $p(1) = 1/10 \quad p(3) = 1/10 \quad p(4) = 2/10$

$p(5) = 4/10$

$A = \{1,3,4\} \longrightarrow p(A) = p(1) + p(3) + p(4)$

$p(A) = 1/10 + 1/10 + 2/10$

$= 4/10$

$p(B) = p(2) + p(4) + p(5) \longrightarrow p(B) = 8/10$

$p(C) = p(1) + p(4) \longrightarrow p(C) = 3/10$

(4)

Ex: $S = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 3, 4, 5, 7\}$ $\quad B = \{2, 4, 6, 3\}$

$p(A) = ?$ $\quad p(B) = ?$

S/n:

$$p(A) = \frac{N(A)}{N(S)} \rightarrow p(A) = \frac{5}{7}$$

$$p(B) = \frac{N(B)}{N(S)} \rightarrow p(B) = \frac{4}{7}$$

Ex: If $A_1, A_2, \dots A_n$ are disjoint events which form a partition of the sample space S

i.e., $A_1 \cup A_2 \dots A_n = S$ $\quad A_i \cap A_j = \phi \quad i, j = 1 \dots n$

Show that for any event B

$$p(B) = p(A_1)\,p(B|A_1) + p(A_2)\,p(B|A_2) + \dots\dots p(A_n)\,p(B|A_n)$$

S/n:

we can write $B = B \cap S$

where putting $S = A_1 \cup A_2 \cup \dots A_n$ we get

$$B = B \cap [A_1 \cup A_2 \cup \dots A_n]$$

$$\Rightarrow B = (B \cap A_1) \cup (B \cap A_2) \cup \dots\dots (B \cap A_n)$$

Since $B \cap A_i$ & $B \cap A_j$ are disjoint events

$$p(B) = p(B \cap A_1) + p(B \cap A_2) + \dots + p(B \cap A_n)$$

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and using $P(B\cap A_i)=P(A_i)P(B|A_i)$

we obtain

$$P(B)=P(A_1)P(B|A_1)+P(A_2)P(B\cap A_2)+\cdots+P(A_n)P(B|A_n)$$

Ex: A box contains m white balls and n black balls. Balls are drawn at random one at a time without replacement. Find the probability of encountering a white ball by the kth draw

Sln: Let W_k denote the event

$W_k=\{$ a white ball is drawn by the kth draw $\}$

$X_i=\{$ i black balls drawn followed by a white ball $\}$

Namely, let $m=50$ $n=50$

consider 27th draw and seeing a white ball at the 27th draw

In total 1 black ball may be drawn before 27th draw
In total 2 black balls may be drawn before 27th draw

In total 26 black balls may be drawn before 27th draw

$W_k=X_0\cup X_1\cup\ldots\cup X_{k-1}$

⑥ Then $p(\omega_k) = \sum_{i=0}^{k-1} p(X_i)$

$p(X_0) = 1 - \frac{n}{m+n}$ $\quad p(X_1) = \frac{n}{m+n} \frac{m}{m+n-1}$

$= \frac{m}{m+n}$

$p(X_2) = \frac{n(n-1) \cdot m}{(m+n)(m+n-1)(m+n-2)}$

$p(\omega_k) = \sum_{i=0}^{k-1} p(X_i)$