

①

Exo

A random variable \tilde{X} takes the values 3, 7, 12

Determine the x -intervals for which cumulative distribution function $F(x)$ is drawn

Slns

$$-\infty < x < 3$$

$$F(x) = \text{Prob}(\tilde{X} \leq x)$$

$$3 \leq x < 7$$

$$7 \leq x < 12$$

$$12 \leq x < \infty$$

Exo

$F(x) = \text{Prob}(\tilde{X} \leq x)$ → cumulative distribution function of \tilde{X}

$$P(x) = \text{Prob}(\tilde{X} = x)$$

$$F(x_i) = \sum_{x \leq x_i} P(x)$$

Numerically verify the above equality

Say that \tilde{X} can take values 1, 2, 3, 4, 5

Let $x_i = 4$

$$F(4) = \text{Prob}(\tilde{X} \leq 4) \\ = \text{Prob}(\tilde{X} = 4) + \text{Prob}(\tilde{X} = 3) + \text{Prob}(\tilde{X} = 2) \\ + \text{Prob}(\tilde{X} = 1)$$

Slns

② $F(4) = \text{Prob}(\hat{X} \leq 4)$ → ^{find} means the prob. that \hat{X} can take values less than or equal to 4

$$F(4) = \text{Prob}(\hat{X}=4) + \text{Prob}(\hat{X}=3) + \text{Prob}(\hat{X}=2) + \text{Prob}(\hat{X}=1)$$

$$\sum_{x \leq 4} p(x) = p(4) + p(3) + p(2) + p(1)$$

$$p(4) = \text{Prob}(\hat{X}=4)$$

$$p(3) = \text{Prob}(\hat{X}=3)$$

$$p(2) = \text{Prob}(\hat{X}=2)$$

$$p(1) = \text{Prob}(\hat{X}=1)$$

③

Ex:

A random variable \tilde{X} can take the values 2, 5, 8

And the prob. mass function $p(x)$ for \tilde{X}

$$p(x) = \text{Prob}(\tilde{X} = x)$$

has the following values

$$p(2) = a \quad p(5) = 2a \quad p(8) = 3a$$

a) Find a

b) Draw cumulative distribution function $F(x)$

Sln:

$$p(x) = \text{Prob}(\tilde{X} = x)$$

$$\sum_x p(x) = 1$$



$$a + 2a + 3a = 1$$

$$6a = 1 \rightarrow a = 1/6$$

$$p(2) = \text{Prob}(\tilde{X} = 2) \rightarrow a$$

$$p(5) = \text{Prob}(\tilde{X} = 5) \rightarrow 2a$$

$$p(8) = \text{Prob}(\tilde{X} = 8) \rightarrow 3a$$

b) \tilde{X} can take the values 2, 5, and 8

- So we first determine the intervals for x using the values that \tilde{X} can take

$$x < 2$$

$$2 \leq x < 5$$

$$5 \leq x < 8$$

$$8 \leq x$$

The intervals for which $F(x)$ will be calculated

④

$$F(x) = \text{Prob}(\tilde{X} \leq x)$$

$$-\infty < x < 2 \rightarrow F(x) = \text{Prob}(\tilde{X} \leq x) = 0$$

$$\begin{aligned} 2 \leq x < 5 &\rightarrow F(x) = \text{Prob}(\tilde{X} \leq x) \\ &= p(2) \\ &= 1/6 \end{aligned}$$

$$\begin{aligned} 5 \leq x < 8 &\rightarrow F(x) = \text{Prob}(\tilde{X} \leq x) \\ &= p(2) + p(5) \\ &= \frac{1}{6} + \frac{2}{6} \\ &= 1/2 \end{aligned}$$

$$\begin{aligned} 8 \leq x < \infty &\rightarrow F(x) = p(2) + p(5) + p(8) \\ &= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} \rightarrow 1 \end{aligned}$$

