

Exercises on Random Variables

Çankaya University

January 7, 2012

1. Let X be a random variable with $f_X(x)$ being uniform in $[-2, 4]$.
 - Find $E[X]$, $Var[X]$, and σ_X .
 - Find $E[X|X > -1]$, $E[X|1 < X \leq 3]$.
 - Let $g(t) = 3t + 4t^2 + 1$. Find $E[g(X)]$.
2. Given random variables X and Y with:

$$f_{XY}(x, y) = \begin{cases} 8xy & \text{if } 0 \leq x \leq 1, y \leq x \\ 0 & \text{o.w.} \end{cases} \quad (1)$$

Are X and Y independent?

3. Let the two dimensional random variables X and Y has the joint pdf given by:

$$f_{XY}(x, y) = \begin{cases} x^2 + \frac{xy}{3} & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{o.w.} \end{cases} \quad (2)$$

- Find the marginal density and distribution functions $f_X(x)$, $f_Y(y)$, $F_X(x)$ and $F_Y(y)$.
 - Evaluate $P(X \geq 0.5, Y \leq 1)$.
4. Show that for a random variable X :
 - $Var[X] = Var[a + X]$, for any constant number a .
 - $Var[cX] = c^2Var[X]$, for any constant number c .
 5. A fair 8-faced die is thrown. If "1", "2", or "3" shows, the casino wins 9000 TL, otherwise the customer wins 4000 TL. Would you like to play this game?

6. Let X be uniform in $[-1, 1]$ and $Y = g(X)$ is given as:

$$g(X) = \begin{cases} X^2 & \text{if } 0 \leq X \\ 0 & \text{o.w.} \end{cases} \quad (3)$$

Find $f_Y(y)$.

7. Given a random variable X and $f_X(x)$ as:

$$f_X(x) = 0.1u(x+1) - 0.1u(x-5) + 0.1\delta(x-1) + 0.3\delta(x-6)$$

where $u(x)$ and $\delta(x)$ are unit-step and Dirac-delta functions, respectively.

- Plot $f_X(x)$ and $F_X(x)$.
- Find $P(3 < X < 6)$.
- Find $P(3 \leq X \leq 6)$.
- Find $P(3 \leq X < 6)$.
- Find $P(3 < X \leq 6)$.
- Find $P(-0.5 \leq X < 1.5)$.
- Find $P(X = -1) + P(X = 1) + P(X = 5) + P(X = 6)$.

8. Let X be a random variable with pdf:

$$f_X(x) = \begin{cases} ax^3 & \text{if } 0 \leq x \leq 3 \\ 0 & \text{o.w.} \end{cases} \quad (4)$$

- Find a .
- Let $B = \{0.1 \leq X \leq 0.4\}$. Find $P(B)$.
- Find $E[X]$ and $Var[X]$.
- Find $E[X|B^c]$.