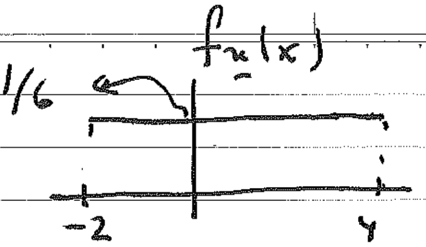
1

$$E(x) = \int_{-2}^{4} x \cdot \frac{1}{6}\, dx$$



$$V(x) = \underbrace{E(x^2)}_{\int_{-2}^{4} x^2 \cdot \frac{1}{6}\, dx} - E^2(x) \Rightarrow \sigma_x = \sqrt{V(x)}$$

$$E(x \mid x > -1) = \int_{-1}^{4} x \cdot \underbrace{f_x(x \mid x > -1)}_{\frac{f_x(x)}{P(x > -1)} = \frac{1/6}{\int_{-1}^{4} \frac{1}{6}\, dx}}\, dx$$

$$E(x \mid 1 < x \leq 3) = \int_{1}^{3} x \cdot \underbrace{f_x(x \mid 1 < x \leq 3)}_{\frac{f_x(x)}{P(1 < x \leq 3)}}\, dx$$

$$E(g(x)) = \int_{-1}^{4} (3x^3 + 4x^2 + 1) \cdot \frac{1}{6}\, dx$$

(2) $$f_{\underset{\sim}{x}\underset{\sim}{y}}(x,y) = \begin{cases} 8xy, & 0 \le x \le 1 \\ & y \le x \\ 0, & \text{o.w.} \end{cases}$$

$$f_{\underset{\sim}{y}}(y) = \int_y^1 8xy\,dx = 4y(1-y^2), \quad 0 \le y \le 1$$

$$f_{\underset{\sim}{x}}(x) = \int_0^x 8xy\,dy = 4x^3, \quad 0 \le x \le 1$$

Check whether $f_{\underset{\sim}{x}}(x) \cdot f_{\underset{\sim}{y}}(y) \stackrel{?}{=} f_{\underset{\sim}{x}\underset{\sim}{y}}(x,y)$

$$4x^3 \cdot 4y(1-y^2) \neq 8xy.$$

So $\underset{\sim}{x}$ & $\underset{\sim}{y}$ are not independent.

(3) $f_{\underset{\sim}{x}}(x) = \int_{-\infty}^{\infty} \left(x^2 + \frac{xy}{3}\right) dy = 2x\left(x + \frac{1}{3}\right)$ for $0 \leq x \leq 1$.

$f_{\underset{\sim}{y}}(y) = \int_{-\infty}^{\infty} \left(x^2 + \frac{xy}{3}\right) dx = \frac{1}{3} + \frac{y}{6}$ for $0 \leq y \leq 2$.

$F_{\underset{\sim}{x}}(x) = \int_{-\infty}^{x} \left(2x'^2 + \frac{2}{3}x'\right) dx'$

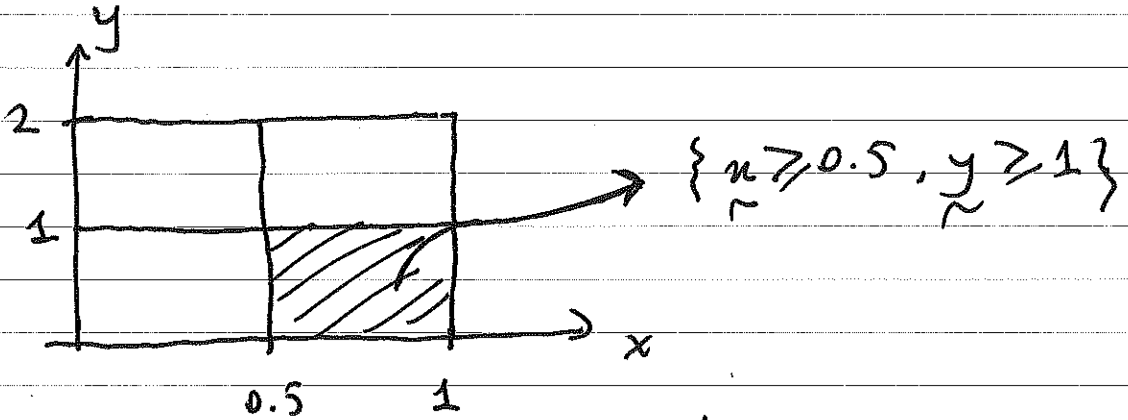
$F_{\underset{\sim}{y}}(y) = \int_{-\infty}^{y} \left(\frac{1}{3} + \frac{y'}{6}\right) dy'$

$$f_{\underset{\sim}{x}}(x) = \begin{cases} 2x\left(x + \frac{1}{3}\right), & 0 \leq x \leq 1 \\ 0, & \text{o.w.} \end{cases}$$

$$f_{\underset{\sim}{y}}(y) = \begin{cases} \frac{1}{3} + \frac{y}{6}, & 0 \leq y \leq 2 \\ 0, & \text{o.w.} \end{cases}$$

$$F_{\underset{\sim}{x}}(x) = \begin{cases} 0, & x < 0 \\ \frac{2}{3}x^3 + \frac{x^2}{3}, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

$$F_Y(y) = \begin{cases} 0 & , \ y < 0 \\ \dfrac{y}{3} + \dfrac{y^2}{12} & , \ 0 \le y \le 2 \\ 1 & , \ y > 2 \end{cases}$$



$$P(\{x \ge 0.5, \ y \ge 1\}) = \int_0^1 \int_{0.5}^1 \left(x^2 + \frac{xy}{3} \right) dx\,dy$$

$$= 11/48.$$

(4) $Var(x) = E(x^2) - E^2(x)$

$$Var(a+x) = E((a+x)^2) - [E(a+x)]^2$$

$$= E(a^2 + 2ax + x^2) - [a + E(x)]^2$$

$$= a^2 + 2aE(x) + E(x^2) - a^2 - 2aE(x) - E^2(x)$$

$$= E(x^2) - E(x^2) = Var(x).$$

$Var(cx) = \ldots$ Do it yourself!

(5) $\Omega = \{1, 2, \ldots, 8\}$, $P(\omega_i) = \frac{1}{8}$.

ω_i

$\underset{\sim}{x}$: earning on a game $\left\{ \underset{\sim}{x} : \Omega \to \{+4000, -9000\} \right.$

$\underset{\sim}{x}$: $-9000, 4000$

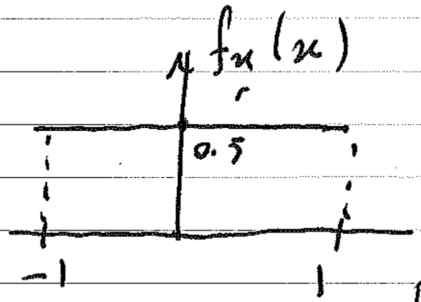
$P(\underset{\sim}{x} = +4000) = P(\{4, \ldots, 8\}) = \frac{5}{8}$.

$P(\underset{\sim}{x} = -9000) = P(\{1, 2, 3\}) = 3/8$.

$E(\underset{\sim}{x}) = 4000 \cdot P(\underset{\sim}{x} = +4000) + (-9000) \cdot P(\underset{\sim}{x} = -9000)$

$\therefore$ Play, if $E(\underset{\sim}{x}) > 0$

(6)



Using the fundamental Thm:

$\underset{\sim}{y} = g(\underset{\sim}{x}) = \underset{\sim}{x}^2$ if $0 \le \underset{\sim}{x}$.

Find the roots of $y = x^2$.

$x_1 = \sqrt{y}$, $x_2 = -\sqrt{y}$.

Consider only $x_1 = \sqrt{y}$, since $\underset{\sim}{x} \geq 0$.

$g'(x) = 2x$, $g'(x_1) = 2\sqrt{y}$.

$$f_Y(y) = \frac{0.5}{|2\sqrt{y}|} \quad \text{for} \quad 0 < y \leq 1.$$

Check whether $\int_0^1 \frac{1}{4\sqrt{y}}\,dy \stackrel{?}{=} 1$.

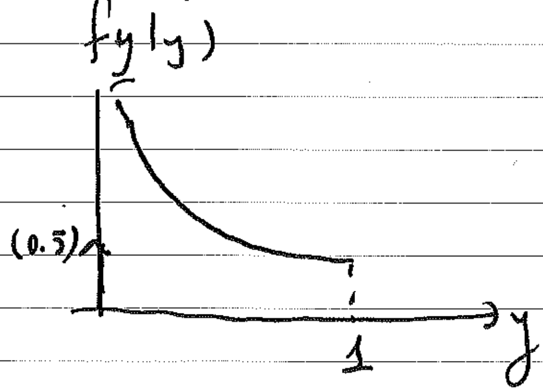
$\frac{1}{2} \neq 1$

$1/2$ is missing!

Hence:

$$f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}}, & 0 < y \leq 1 \\ 0.5\,\delta(y), & y = 0 \end{cases}$$

$F_X(0) \neq 0$



(7) + (8) Do them yourself!