

Cankaya University

ECE307 - Quiz-3

October 17th-21st, 2011

1. A die is weighted so that the outcomes produce the following probability distribution.

Outcome	1	2	3	4	5	6
Probability	0.2	0.4	0.1	0.1	0.1	0.1

Consider the events:

$A = \{\text{odd number}\}$, $B = \{2,3,4,5\}$, $C = \{x|x < 4\}$, and $D = \{x|x > 6\}$.

Find $P(A)$, $P(B)$, $P(C)$, $P(D)$.

Solution:

$P(A) = P(\{1, 3, 5\}) = P(\{1\}) + P(\{3\}) + P(\{5\})$ (Since, these singletons are disjoint sets, their union's probability is equal to their sum - Ax.3).

$$P(A) = 0.2 + 0.1 + 0.1 = 0.4$$

Similarly, $P(B) = 0.4 + 0.1 + 0.1 + 0.1 = 0.7$, $P(C) = 0.7$ and $P(D) = 0$.

2. Assume that each day of the year is equally probable for a birthday and that one year is 365 days. In a group of 28 students, find the probability that at least two students have the same birthday.

Solution:

Let $A = \{\text{At least two students have the same birthday}\}$.

Then $A^c = \{\text{Each student has a unique birthday}\}$.

Note that $P(A^c) = \frac{365 \cdot 364 \cdots (365 - 28 + 1)}{365^{28}}$.

Hence, $P(A) = 1 - P(A^c)$.

3. Consider $\Omega = \{a, b, c, d\}$ with equally likely elements. Consider the events:
 $A = \{a, c\}$, $B = \{b, c\}$, and $C = \{d, c\}$.

a) Show that A, B and C are pairwise independent.

b) Show that A, B and C are not independent.

Solution:

a) Check whether $P(A \cap B) \stackrel{?}{=} P(A)P(B)$.

$A \cap B = \{c\}$, hence $P(A \cap B) = \frac{1}{4}$.

$P(A)P(B) = \frac{1}{2} \cdot \frac{1}{2} = 0.25$. Since, $P(A \cap B) \neq P(A)P(B)$, A and B are independent events.

You can prove for A and C, and B and C, in a similar fashion.

b) On top of the pair-wise independence, which is already checked in a), one must check whether $P(A \cap B \cap C) \stackrel{?}{=} P(A)P(B)P(C)$.

$A \cap B \cap C = \{c\}$, hence $P(A \cap B \cap C) = \frac{1}{4}$.

On the other hand, $P(A)P(B)P(C) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 0.125$.

Since, $P(A \cap B \cap C) \neq P(A)P(B)P(C)$, A and B and C are not independent.