

① We toss a fair coin two times.

$S = \{hh, ht, th, tt\}$ \rightarrow Sample space of the experiment

s_i \rightarrow denotes a simple outcome

We define the random variable \tilde{X} on simple outcomes as follows

$$\tilde{X}(s_i) = \left\{ \begin{array}{l} 2 \times \text{number of heads} - 1 \times \text{number of tails} \\ \text{in } s_i \end{array} \right\}$$

a) Find all values of the random variable \tilde{X}

b) Find the subsets

$$\Leftrightarrow \{s_i \mid \tilde{X}(s_i) = 1\}$$

$$\Leftrightarrow \{s_i \mid \tilde{X}(s_i) = -2\}$$

$$\Leftrightarrow \{s_i \mid \tilde{X}(s_i) \leq 1\}$$

c) Find the prob. mass function

$$p(x) = \text{Prob}(\tilde{X} = x)$$

Draw the graph of $p(x)$

d) $F(x) = \text{Prob}(\tilde{X} \leq x)$

Find cumulative distribution function $F(x)$
and draw the graph of $F(x)$