

Friday, November 20, 2015

ECE-307, Probability and Random Variables, Fall 2015-106
Midterm Exam Questions

1) (15pts) Using probability law axioms show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2) (5pts) For a biased coin $P(h) = 1/3$, $P(t) = 2/3$ and for a three sided die $P(f_1) = \frac{1}{4}$, $P(f_2) = \frac{1}{4}$, $P(f_3) = \frac{1}{2}$. We flip the coin and toss the die at the same time. Write the sample space and indicate the probabilities of the simple outcomes.

3) (10pts) A fair coin is tossed 10 times. What is the probability that the number of times "3" shows up is a digit between 3 and 7.

4) (15pts) A company has three factories 1, 2, and 3 that produce the same chip, each producing 20%, 50%, and 30% of the total production. The probability of a defective chip at 1, 2, 3 is 0.01, 0.05, and 0.02 respectively. Suppose that someone shows us a defective chip. What is the probability that the defective chip comes from factory 1.

5) (15pts) For a communication system the following information is given

The source produces 0 with probability $p=0.3$

The source produces 1 with probability $q=0.7$

$r \rightarrow$ Denotes the received symbol

$r \rightarrow$ Denotes the transmitted symbol

$$P(r = 0 | s = 0) = \epsilon_0 \quad P(r = 1 | s = 0) = 1 - \epsilon_0$$

$$P(r = 1 | s = 1) = \epsilon_1 \quad P(r = 0 | s = 1) = 1 - \epsilon_1$$

a) Calculate the probability of not receiving a transmitted symbol correctly

b) Calculate the probability of receiving a symbol correctly

c) If the received symbol is $r = 0$ then what is the probability that the transmitted symbol is $s = 0$

6) (15pts) $S = \{-2, -1, 1, 2, 3\} \rightarrow$ is the sample space of an experiment. The random variable \tilde{X} on this sample space is defined as

$$\tilde{X}(\rho) = \begin{cases} \rho^2 - 2 & \text{if } |\rho| \text{ is odd} \\ \rho^2 + 2 & \text{if } |\rho| \text{ is even} \end{cases} \quad \rho \in S$$

Find the probability mass function of \tilde{X} and draw it. Draw the cumulative distribution function $F(x)$ of RV \tilde{X} . Calculate $Prob\{\tilde{X} > 4\}$ and $Prob\{\tilde{X} \leq 6\}$

7) (20pts) Probability mass function of random variable \tilde{X} is defined as

$$p(x) = \begin{cases} a & \text{if } x = 1 \text{ or } x = -1 \\ 2a & \text{if } x = 2 \end{cases}$$

a) Find a b) Draw the cumulative distribution function of \tilde{X} , i.e. $F(x)$ c) Calculate the mean value and variance of \tilde{X} d) If $\tilde{Y} = 2\tilde{X} + 1$ Find the mean value and variance of \tilde{Y}

Good Luck O.G.