Saturday, November 27, 2010

ECE 307 Midterm I

1) (10pts) Two biased coins are tossed. For the first coin $\text{Prob}(h_1)=\frac{1}{3}$, $\text{Prob}(t_1)=\frac{2}{3}$, for the second coin $\text{Prob}(h_2)=\frac{2}{3}$, $\text{Prob}(t_2)=\frac{1}{3}$. Find the followings.

   a) Sample Space? b) Probability of simple events? c) Probability of events with 3 elements? d) $A=\{\text{at least one biased tail}\}$, Probability of (A)=?

2) (10pts) Sample space of an experiment is $S=\{a, b, c, d\}$. Probability of simple events are $\text{Prob}(a)=\frac{1}{16}$, $\text{Prob}(b)=\frac{4}{16}$, $\text{Prob}(c)=\frac{5}{16}$, $\text{Prob}(d)=\frac{6}{16}$. Given the events $A=\{a, b, c\}$, $A=\{b, c, d\}$ determine the followings

   a) $\text{Prob}(A \mid B)=?$ b) A and B are independent or not? c) A and B are conditionally independent or not?

3) (10pts) You enter a chess tournament. There are three chess teams. Team-1 consists of 20 players. Team-2 consists of 20 players and team-3 consists of 10 payers. Your prob. of winning against a player in team-1 is 0.5, and it is 0.3 and 0.6 for the team-2, team-3 respectively.

   You play against a randomly chosen opponent. What is the probability of your winning?

4) (10pts) You play 100 chess games. The prob. of loosing a chess game is 0.25. What the prob. that the number of games you win is between 60 and 80.

5) (10pts) How many distinct combinations of the following letter sequences are available?
   a) AABBCDDDEF
   b) ABCDEFFGGH

6) (10pts) Experiment: A fair coin is tossed 3 times.
   $s_i \rightarrow$ Simple Events. Random variable is defined as

   $$\tilde{X}(s_i)=\{2\times \text{number of heads in } s_i \text{ - number of tails in } s_i\}$$

   Determine the followings sets.

   a) $\tilde{X}=3$ b) $\tilde{X}=0$ c) $\tilde{X}\le 3$

   Compute the following probabilities

   $\text{Prob}(\tilde{X}=3)$?, $\text{Prob}(\tilde{X}<3\cap\tilde{X}=0)$

7) (20pts) Experiment: A fair coin is tossed 2 times.
$s_i \rightarrow$ Simple Events. Random variable is defined as

$$\tilde{X}(s_i) = \{2 \times \text{number of tails in } s_i\}$$

Determine $p(x) \rightarrow$ Probability density function of $\tilde{X}$. Draw $p(x)$.

8) (20pts) Experiment: A biased coin with $\text{Prob}(h_b) = \frac{1}{3}$, $\text{Prob}(t_b) = \frac{2}{3}$, is tossed 2 times.
$s_i \rightarrow$ Simple Events. Random variable is defined as

$$\tilde{X}(s_i) = \{2 \times \text{number of biased heads in } s_i \text{ - number of biased tails in } s_i\}$$

Determine $F(x) \rightarrow$ Cumulative probability distribution function of $\tilde{X}$. Draw $F(x)$.

Best LUCK. **O._G.**

(A)

ECE 307 midterm
Solutions

1) $P(h_1)=1/3 \quad P(t_1)=2/3 \quad P(h_2)=2/3 \quad P(t_2)=1/3$

a) Sample Space $= \{h_1h_2, h_1t_2, h_2t_1, t_1t_2\}$

b) $P(h_1h_2)=2/9 \quad P(h_1t_2)=1/9 \quad P(h_2t_1)=4/9$

$P(t_1t_2)=2/9$

c) Simple events with 2 elements. → 2 element Subsets of Sample space

$A_1=\{h_1h_2, h_1t_2, t_1h_2\} \rightarrow P(A_1)=2/9+1/9+4/9 = 7/9$

$A_2=\{h_1h_2, h_1t_2, t_1t_2\} \rightarrow P(A_2)=5/9$

$A_3=\{h_1t_2, h_2t_1, t_1t_2\} \rightarrow P(A_3)=3/9$

$A_4=\{h_1t_2, h_2t_1, t_2t_1\} \rightarrow P(A_4)=8/9$

d) $A=\{$at least one biased toss$\}$

$A=\{h_1h_2, h_1t_2, h_2t_1\}$

$P(A)=P(h_1h_2)+P(h_1t_2)+P(h_2t_1)$

$= 7/9$

2) a) $\text{Prob}(A|B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{P(\{b,c\})}{P\{b,c,d\}}=\dfrac{4/16+5/16}{4/16+5/16+6/16}$

$=\dfrac{9}{15}=3/5$

(1)

(2) b) $P(A \cap B) \overset{?}{=} P(A)\,P(B)$

$\downarrow$

$9/16 \neq \frac{10}{16} \cdot \frac{15}{16}$

A & B are not independent

c) Try to find C

and show that

$P(A \cap B \mid C) \neq P(A \mid C) \cdot P(B \mid C)$

$C = \{a, c, d\}$ → can be found

A & B are not cond. independent.

OR use $P(A \mid C) = P(A \mid B \cap C)$

(3) Total number of players $20 + 20 + 10 = 50$

Prob of playing against to team 1

If $P_{T1} = 20/50$

against to team 2 is $P_{T2} = 20/50$

" " " 3 is $P_{T3} = 10/50$

$$P_{rob}(\text{winning}) = P(w \mid T_1) P_{T1} + P(w \mid T_2) P_{T2} + P(w \mid T_3) P_{T3}$$

$$= 0.5 \cdot \frac{2}{5} + 0.3 \cdot \frac{2}{5} + 0.6 \cdot \frac{1}{5}$$

C

(4) $P = \sum_{k=60}^{80} \binom{100}{k} (0.75)^k (0.25)^{100-k}$

(5) a) $\overbrace{AA}^{2}\ \overbrace{BB}^{2}\ C\ \overbrace{DDD}^{3}\ EF \rightarrow \underbrace{\text{10 letters}} \rightarrow \frac{10!}{2!\,2!\,3!}$

b) $A\ BCDE\ \overbrace{FF}\ \overbrace{GG}\ H \rightarrow \frac{10!}{2!\,2!}$

(6) $S = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$

→ sample space

$\hat{X}(hhh) = 6$
$\hat{X}(hht) = 3$
$\hat{X}(hth) = 3$
$\hat{X}(htt) = 0$
$\hat{X}(thh) = 3$
$\hat{X}(tht) = 0$
$\hat{X}(tth) = 0$
$\hat{X}(ttt) = -3$

a) $\hat{X}=3 \rightarrow \{s_i \mid \hat{X}(s_i)=3\} \rightarrow \{hht, hth, thh\}$

b) $\hat{X}=0 \rightarrow \{s_i \mid \hat{X}(s_i)=0\} \rightarrow \{htt, tht, tth\}$

c) $\hat{X}<3 \rightarrow \{s_i \mid \hat{X}(s_i)<3\} \rightarrow \{S - \{hhh\}\}$

b) $Prob(\hat{X}=3) = 1/8$ $\quad Prob(\hat{X}<3 \cap \hat{X}=0) = Prob(\hat{X}=0) = 3/8$

(D)

(7) $S = \{hh, ht, th, tt\}$

$X(hh) = 0$

$X(ht) = 2$

$X(th) = 2$

$X(tt) = 4$

$p(x) = Prob(X = x)$

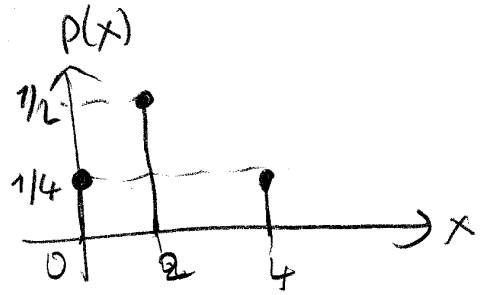
$x = 0 \rightarrow p(0) = Prob(X = 0)$

$= Prob(hh) = 1/4$

$x = 2 \rightarrow p(2) = 2/4 = 1/2$

$x = 4 \quad p(4) = 1/4$

$$p(x) = \begin{cases} 1/4 & x = 0, 4 \\ 1/2 & x = 2 \end{cases}$$



(8) $S = \{h_b h_b, h_b t_b, t_b h_b, t_b t_b\}$

$X(h_b h_b) = 4$

$X(h_b t_b) = 1$

$X(t_b h_b) = 1$

$X(t_b t_b) = -2$

$-\infty < x < -2 \rightarrow F(x) = P(X \leq x)$

$-2 \leq x < 1 \rightarrow F(x) = P(X \leq x)$

$1 \leq x < 4 \rightarrow F(x) = P(X \leq x)$

$4 \leq x < \infty \rightarrow F(x) = P(X \leq x)$

(E)

$-\infty < x < -2 \rightarrow F(x) = P(\tilde{x} \leq x)$

$= P(\phi) = 0$

$-2 \leq x < 1 \rightarrow F(x) = P(\tilde{x} \leq x)$

$= P(t_b t_b) = 4/9$

$1 \leq x < 4 \rightarrow F(x) = P(\tilde{x} \leq x)$

$= P(t_b t_b, t_b h_b, h_b t_b)$

$= 1 - P(h_b h_b)$

$= 1 - 1/9$

$= 8/9$

$4 \leq x < \infty \rightarrow F(x) = 1$